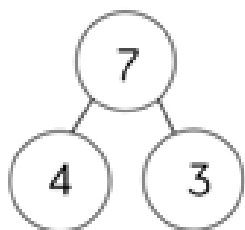




Mytham Calculation Policy

Pictorial Representations

Representations are ways of demonstrating to the children the composition of number and may help to give a framework to understanding the 4 calculations.



For example; a part whole model shows children the composition of a number.

$$3 + 4 = 7$$

$$4 + 3 = 7$$

$$7 - 3 = 4$$

$$7 - 4 = 3$$

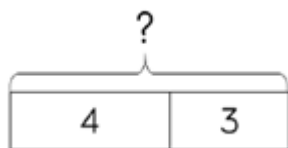
For example; numicon shows children the composition of a number. These basic number facts provide a basis for fundamental maths skills.

$$3 + 4 = 7$$

$$4 + 3 = 7$$

$$7 - 3 = 4$$

$$7 - 4 = 3$$



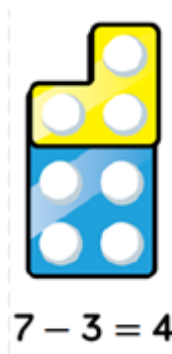
For example; Bar modelling again shows the children the composition of a number. All of which promote maths fluency in the early stages.

$$3 + 4 = 7$$

$$4 + 3 = 7$$

$$7 - 3 = 4$$

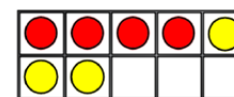
$$7 - 4 = 3$$



Pictorial Strategies

Strategies provide children with a way of completing one of the four operations, this can begin with concrete resources and then progress to abstract written strategies.

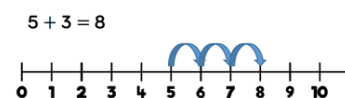
Tens frames are a practical example of a strategy



$$\begin{array}{l} 4 + 3 = 7 \\ 3 + 4 = 7 \\ 7 - 3 = 4 \\ 7 - 4 = 3 \end{array}$$

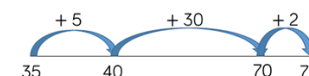
4 is a part.
3 is a part.
7 is the whole.

Numberlines

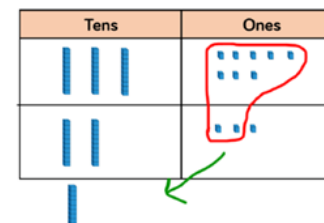


$$5 + 3 = 8$$

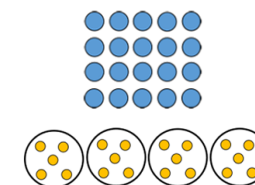
$$35 + 37 = 72$$



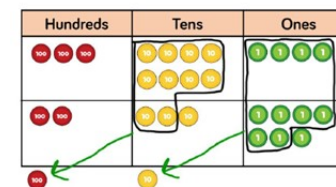
Base ten which transpires into tens and ones written



Arrays/sharing/grouping



Thousands, tens, ones which transpires into column addition/subtraction



$$\begin{array}{r} 384 \\ + 237 \\ \hline 621 \\ 11 \end{array}$$

Abstract: Formal written methods

This includes; column addition/subtraction, short multiplication, short division (bus stop), Long multiplication, long division

Welcome to the Mytham Mathematics Calculation Policy.

This document is broken down into addition and subtraction, and multiplication and division.

At the start of this policy, there is an overview of the different models and images that can support the teaching of different concepts. These provide explanations of the benefits of using the models and show the links between different operations.

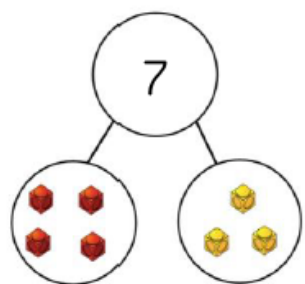
Skill: Add three 1-digit numbers		Year: 2
	$7 + 6 + 3 = 16$	When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports children in their understanding of commutativity. Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

Each operation is then broken down into skills and each skill has a dedicated page showing the different models and images that could be used to effectively teach that concept.

There is an overview of skills linked to year groups to support consistency through out school. A glossary of terms is provided at the end of the calculation policy to support understanding of the key language used to teach the four operations.

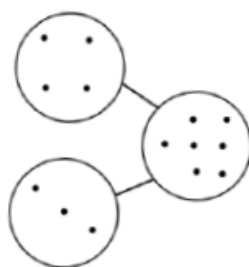
Benefits	
	This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model. When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total. When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part. Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns. In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Part-Whole Model



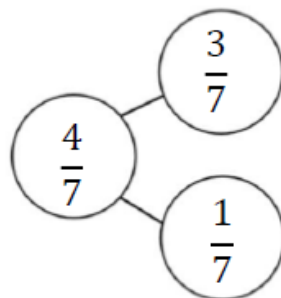
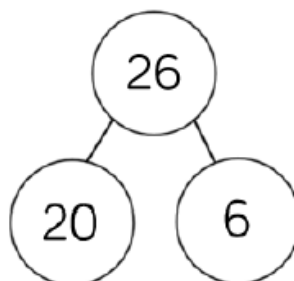
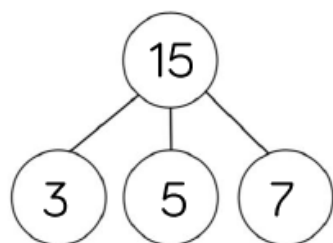
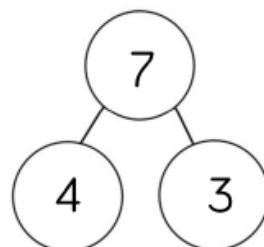
$$7 = 4 + 3$$

$$7 = 3 + 4$$



$$7 - 3 = 4$$

$$7 - 4 = 3$$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

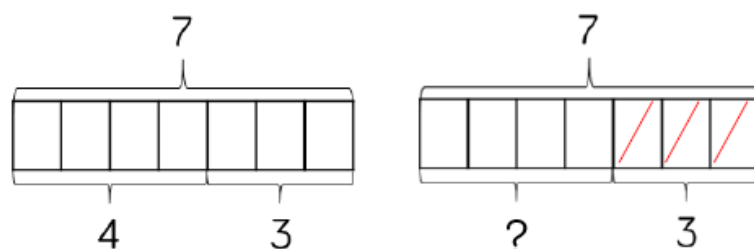
In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)

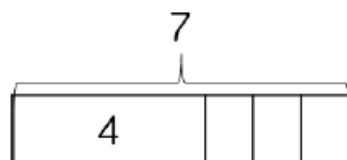
Concrete



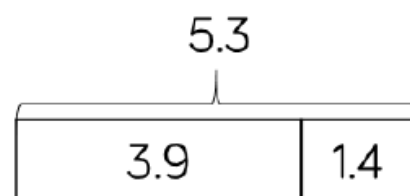
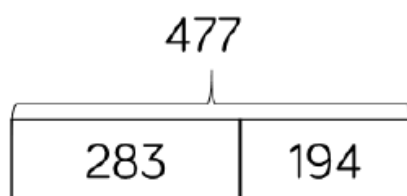
Discrete



Combination



Continuous



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

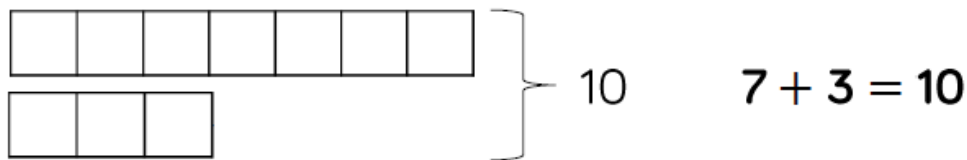
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

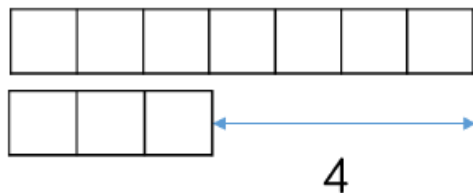
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)

Discrete

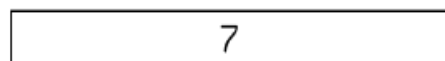


$$7 + 3 = 10$$

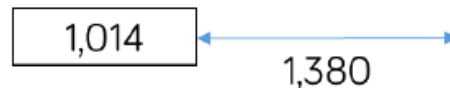
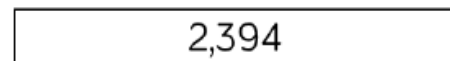


$$7 - 3 = 4$$

Continuous



$$7 - 3 = 4$$



$$2,394 - 1,014 = 1,380$$

Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Numicon



$$4 + 6 = 10$$

$$3 + 6 = 9$$

$$7 - 3 = 4$$

Benefits

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

Cubes



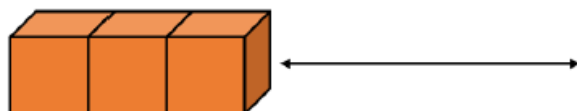
$$7 = 4 + 3$$



$$7 = 3 + 4$$



$$7 - 3 = 4$$



$$7 - 3 = 4$$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

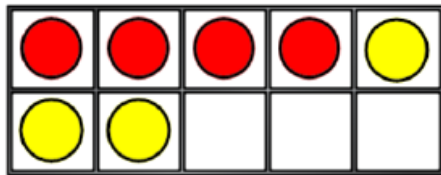
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

Ten Frames (within 10)



$$4 + 3 = 7$$

$$3 + 4 = 7$$

$$7 - 3 = 4$$

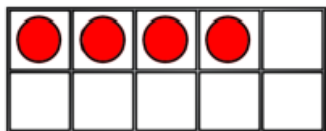
$$7 - 4 = 3$$

4 is a part.

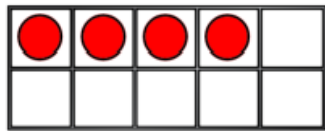
3 is a part.

7 is the whole.

First

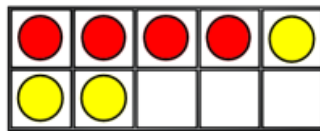


Then

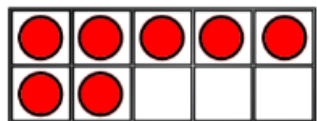


$$4 + 3 = 7$$

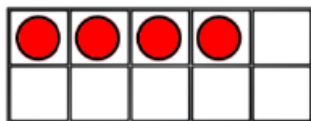
Now



First

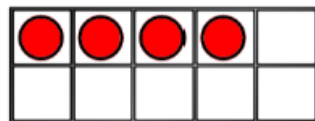


Then



$$7 - 3 = 4$$

Now



Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

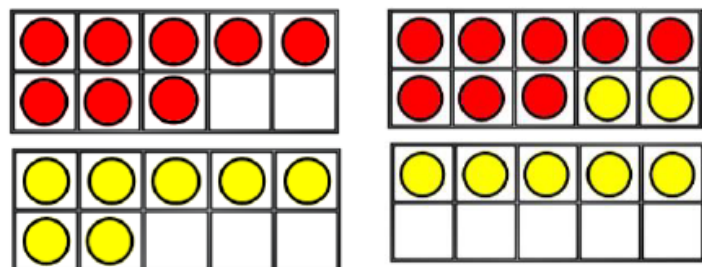
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts.

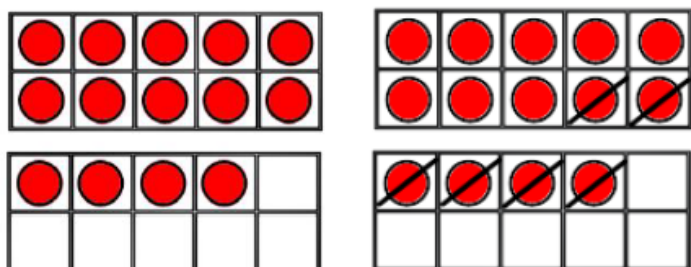
Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

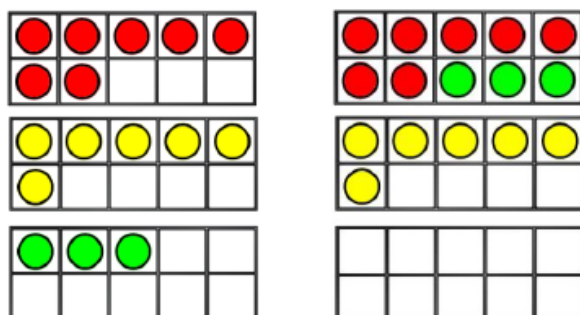
Ten Frames (within 20)



$$\begin{array}{r} 8 + 7 = 15 \\ \swarrow \searrow \\ 2 \quad 5 \end{array}$$



$$\begin{array}{r} 14 - 6 = 8 \\ \swarrow \searrow \\ 4 \quad 2 \end{array}$$



$$\begin{array}{r} 7 + 6 + 3 = 16 \\ \swarrow \quad \searrow \\ \quad 10 \end{array}$$

Benefits

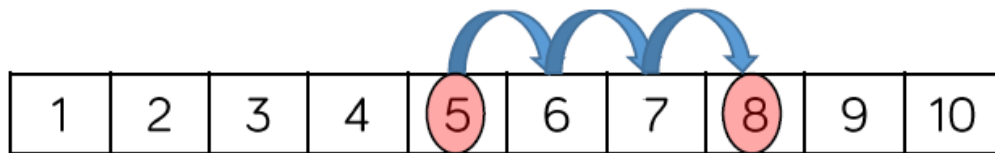
When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

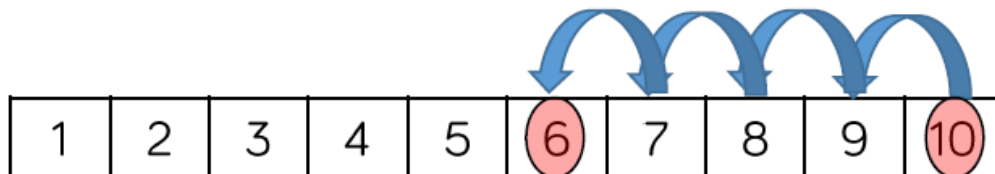
When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Number Tracks

$$5 + 3 = 8$$



$$10 - 4 = 6$$



$$8 + 7 = 15$$



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

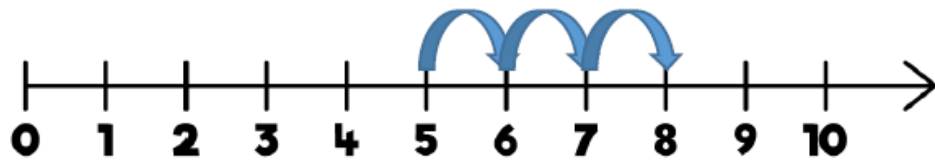
When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

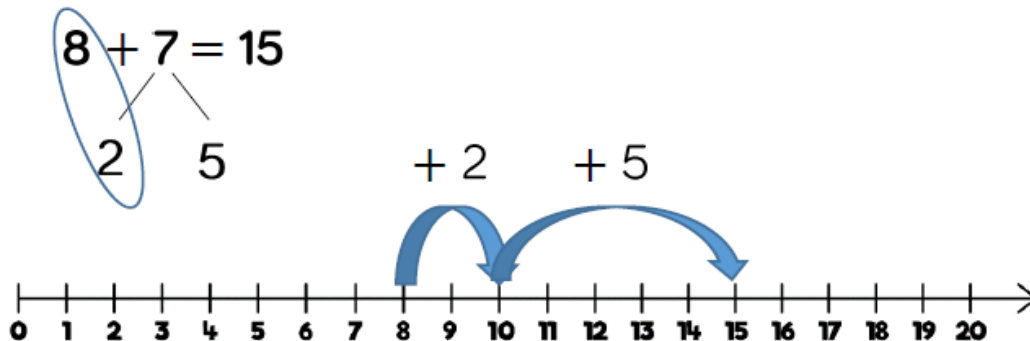
Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

Number Lines (labelled)

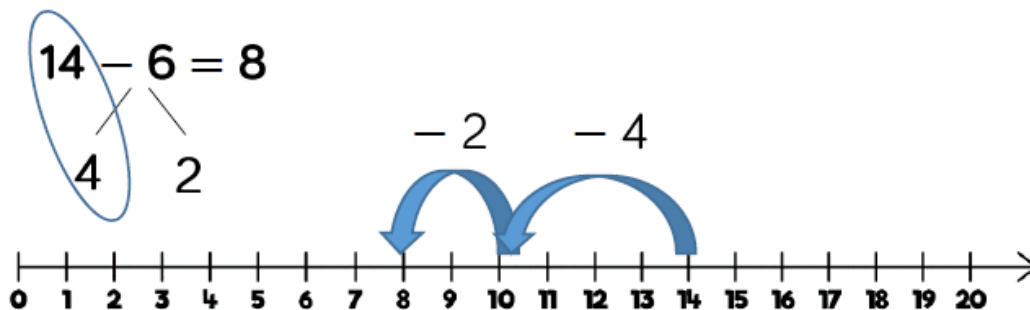
$$5 + 3 = 8$$



$$8 + 7 = 15$$



$$14 - 6 = 8$$



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

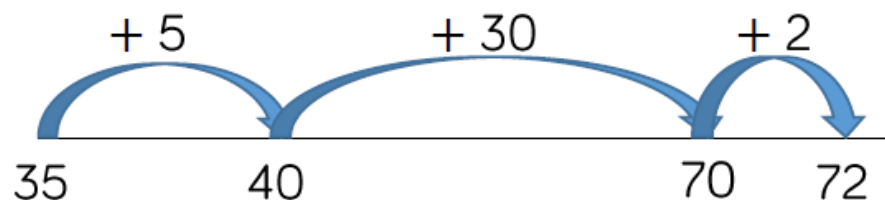
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

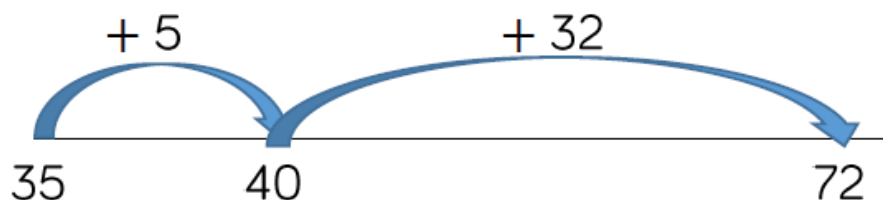
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Number Lines (blank)

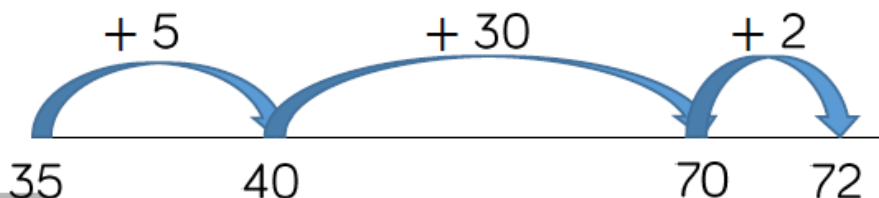
$$35 + 37 = 72$$



$$35 + 37 = 72$$



$$72 - 35 = 37$$



Benefits

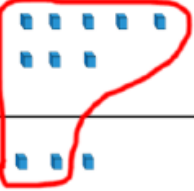
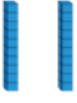
Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

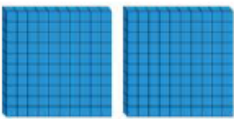
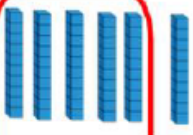
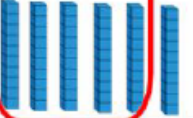
Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Base 10 (addition)

Tens	Ones
	
	



$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ \hline 1 \end{array}$$

Hundreds	Tens	Ones
		
		



$$\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ \hline 1 \end{array}$$

Benefits

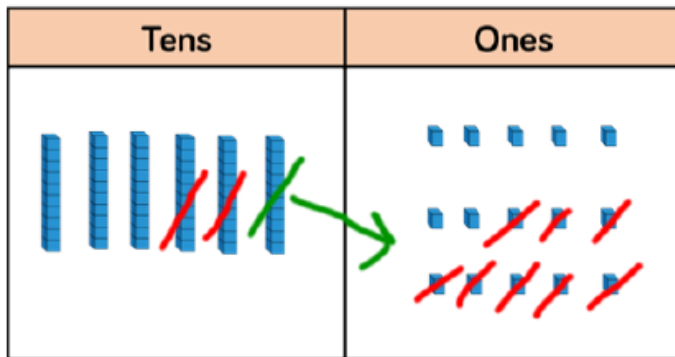
Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

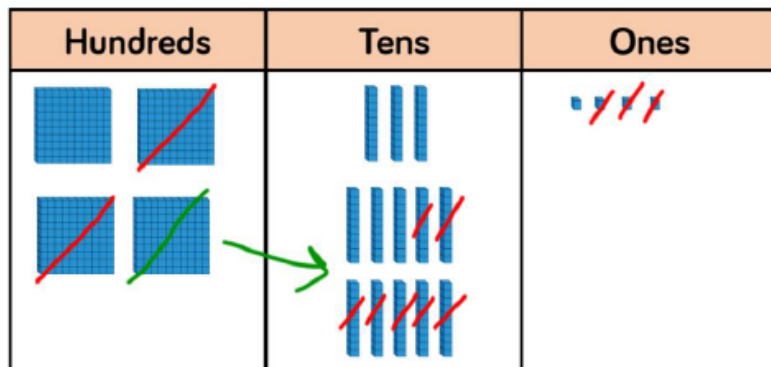
When adding, always start with the smallest place value column. Here are some questions to support children.

- How many ones are there altogether?
- Can we make an exchange? (Yes or No)
- How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
- How many ones do we have left? (Write in ones column)
- Repeat for each column.

Base 10 (subtraction)



$$\begin{array}{r} 5 \quad 1 \\ 65 \\ - 28 \\ \hline 37 \end{array}$$



$$\begin{array}{r} 3 \quad 1 \\ 435 \\ - 273 \\ \hline 262 \end{array}$$

Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Place Value Counters (addition)

Hundreds	Tens	Ones
100 100 100	10 10 10 10 10 10 10 10	1 1 1 1
100 100	10 10 10	1 1 1 1 1 1 1

100 10

$$\begin{array}{r} 384 \\ + 237 \\ \hline 621 \\ 11 \end{array}$$

Ones	Tenths	Hundredths
1 1 1	0.1 0.1 0.1 0.1 0.1 0.1	0.01 0.01 0.01 0.01 0.01
1 1	0.1 0.1 0.1 0.1	0.01

1

$$\begin{array}{r} 3.65 \\ + 2.41 \\ \hline 6.06 \\ 1 \end{array}$$

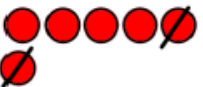
Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Place Value Counters (Subtraction)

Hundreds	Tens	Ones
		

$$\begin{array}{r} 652 \\ - 207 \\ \hline 445 \end{array}$$

Thousands	Hundreds	Tens	Ones
			

$$\begin{array}{r} 4357 \\ - 2735 \\ \hline 1622 \end{array}$$

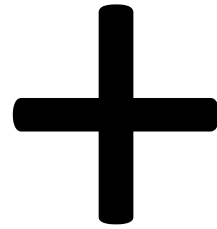
Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

Addition



Addition representations used in Year 1 & 2

Skill	Year	Representations and models	
Add two 1-digit numbers to 10	1	Part-whole model Bar model Numicon	Ten frames (within 10) Number tracks
Add 1 and 2-digit numbers to 20	1	Part-whole model Bar model Numicon Ten frames (within 20)	Number tracks Number lines (labelled)
Add three 1-digit numbers	2	Part-whole model Bar model	Ten frames (within 20) Numicon
Add 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Hundred square
Add two 2-digit numbers	2	Part-whole model Bar model Number lines (blank)	Base 10 Place value counters

Addition representations used in Year 1 & 2

Skill: Add 1-digit numbers within 10	Year: 1
4 + 3 = 7	When adding numbers to 10, children can explore both aggregation and augmentation. The part-whole model, discrete and continuous bar model, number shapes and ten frame support aggregation. The combination bar model, ten frame, bead string and number track all support augmentation.

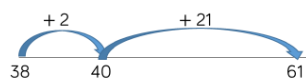
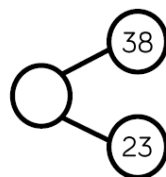
Skill: Add three 1-digit numbers	Year: 2
7 + 6 + 3 = 16	When adding three 1-digit numbers, children should be encouraged to look for number bonds to 10 or doubles to add the numbers more efficiently. This supports children in their understanding of commutativity. Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.

Skill: Add 1 and 2-digit numbers to 20	Year: 1/2
8 + 7 = 15	When adding one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. Different manipulatives can be used to represent this exchange. Use concrete resources alongside number lines to support children in understanding how to partition their jumps.

Addition representations used in Year 1 & 2

Skill: Add two 2-digit numbers to 100

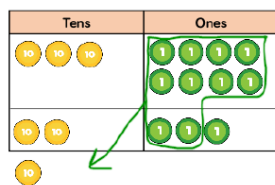
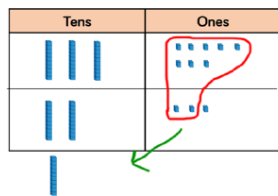
Year: 2/3



?

38	23
----	----

$$38 + 23 = 61$$



At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Children can also use a blank number line to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient.

Please note: column addition not taught until Year 3.

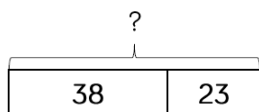
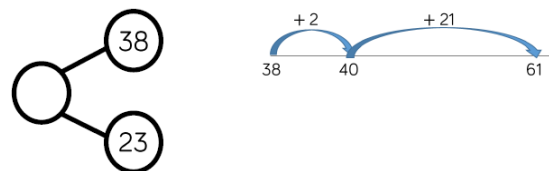
Addition representations used in Year 3 & 4

Skill	Year	Representations and models	
Add with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column addition
Add with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column addition

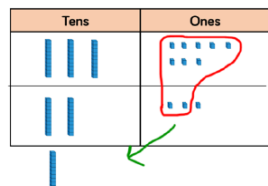
Addition representations used in Year 3 & 4

Skill: Add two 2-digit numbers to 100

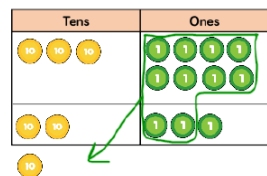
Year: 2/3



$$38 + 23 = 61$$



$$\begin{array}{r} 38 \\ + 23 \\ \hline 61 \\ 1 \end{array}$$

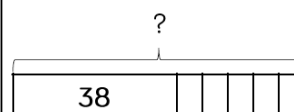
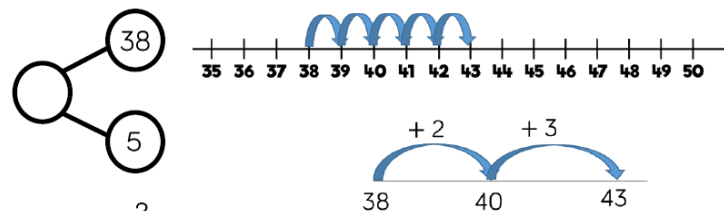


At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient.

Children can also use a blank number line to count on to find the total. Encourage them to jump to multiples of 10 to become more efficient.

Skill: Add 1-digit and 2-digit numbers to 100

Year: 2/3



$$38 + 5 = 43$$

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

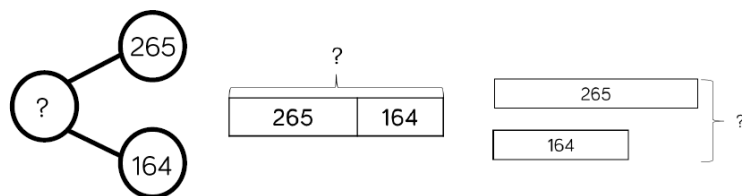
When adding single digits to a two-digit number, children should be encouraged to count on from the larger number.

They should also apply their knowledge of number bonds to add more efficiently e.g. $8 + 5 = 13$ so $38 + 5 = 43$.

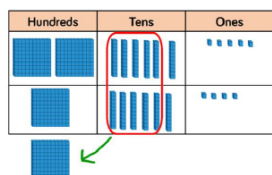
Hundred squares and straws can support children to find the number bond to 10.

Skill: Add numbers with up to 3 digits

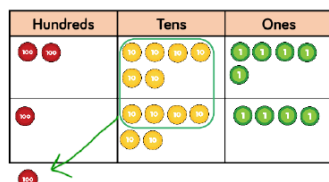
Year: 3



$$265 + 164 = 429$$



$$\begin{array}{r} 265 \\ + 164 \\ \hline 429 \\ 1 \end{array}$$



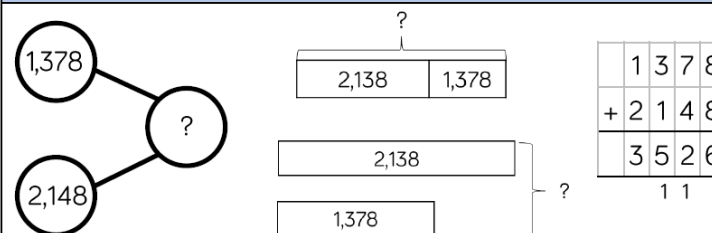
Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 3 digits.

Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

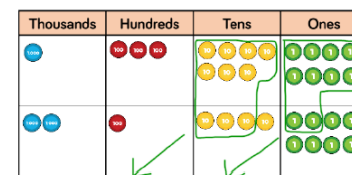
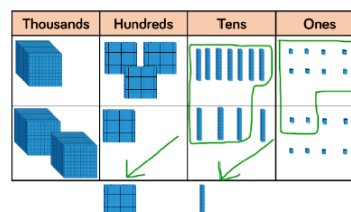
Plain counters on a place value grid can also be used to support learning.

Skill: Add numbers with up to 4 digits

Year: 4



$$1,378 + 2,148 = 3,526$$



Base 10 and place value counters are the most effective manipulatives when adding numbers with up to 4 digits.

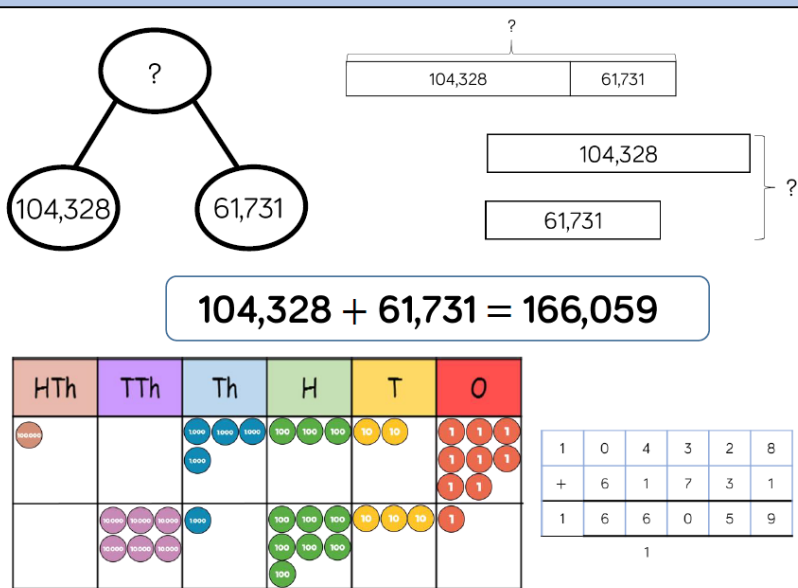
Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method.

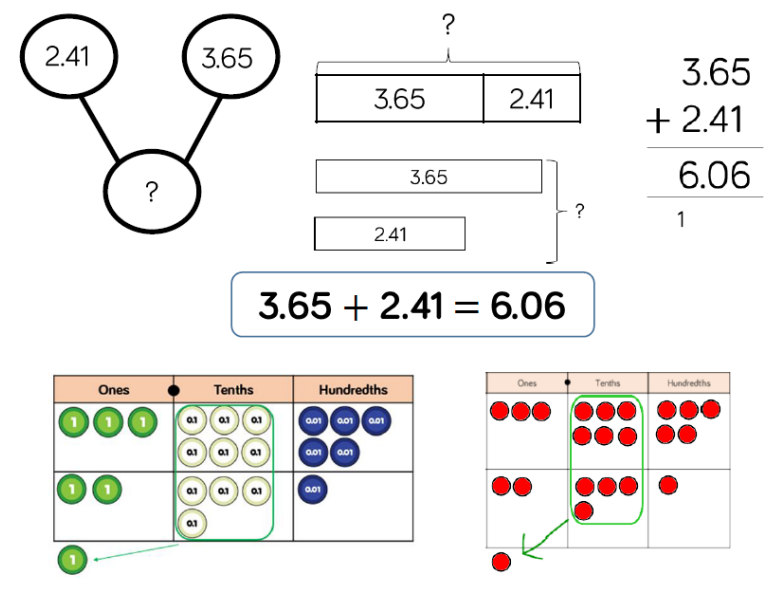
Plain counters on a place value grid can also be used to support learning.

Addition representations used in Year 5 & 6

Skill	Year	Representations and models	
Add with more than 4 digits	5	Part-whole model Bar model	Place value counters Column addition
Add with up to 3 decimal places	5	Part-whole model Bar model	Place value counters Column addition

Addition representations used in Year 5 & 6

Skill: Add numbers with more than 4 digits		Year: 5/6																		
 The diagram illustrates the addition of 104,328 and 61,731. At the top, a tree diagram shows a question mark branching into 104,328 and 61,731. Below this, the numbers are shown in boxes, with a bracket indicating their sum. A large box contains the equation: $104,328 + 61,731 = 166,059$. To the right, a place value grid is shown with columns for HTh, TTh, Th, H, T, and O. The grid contains counters representing the digits of the two numbers being added. Below the grid, a column method calculation is shown: <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>1</td><td>0</td><td>4</td><td>3</td><td>2</td><td>8</td></tr> <tr><td>+</td><td>6</td><td>1</td><td>7</td><td>3</td><td>1</td></tr> <tr><td>1</td><td>6</td><td>6</td><td>0</td><td>5</td><td>9</td></tr> </table> 1		1	0	4	3	2	8	+	6	1	7	3	1	1	6	6	0	5	9	Place value counters or plain counters on a place value grid are the most effective concrete resources when adding numbers with more than 4 digits. At this stage, children should be encouraged to work in the abstract, using the column method to add larger numbers efficiently.
1	0	4	3	2	8															
+	6	1	7	3	1															
1	6	6	0	5	9															

Skill: Add with up to 3 decimal places		Year: 5											
 The diagram illustrates the addition of 3.65 and 2.41. At the top, a tree diagram shows a question mark branching into 3.65 and 2.41. Below this, the numbers are shown in boxes, with a bracket indicating their sum. A large box contains the equation: $3.65 + 2.41 = 6.06$. To the right, a place value grid is shown with columns for Ones, Tenths, and Hundredths. The grid contains counters representing the digits of the two numbers being added. Below the grid, a column method calculation is shown: <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><td>3</td><td>6</td><td>5</td></tr> <tr><td>+</td><td>2</td><td>4</td><td>1</td></tr> <tr><td>6</td><td>0</td><td>6</td><td></td></tr> </table> 1		3	6	5	+	2	4	1	6	0	6		Place value counters and plain counters on a place value grid are the most effective manipulatives when adding decimals with 1, 2 and then 3 decimal places. Ensure children have experience of adding decimals with a variety of decimal places. This includes putting this into context when adding money and other measures.
3	6	5											
+	2	4	1										
6	0	6											

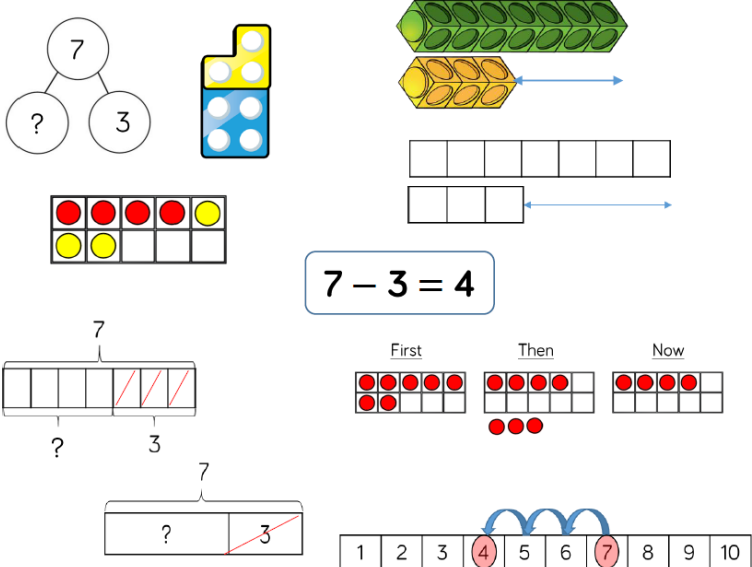
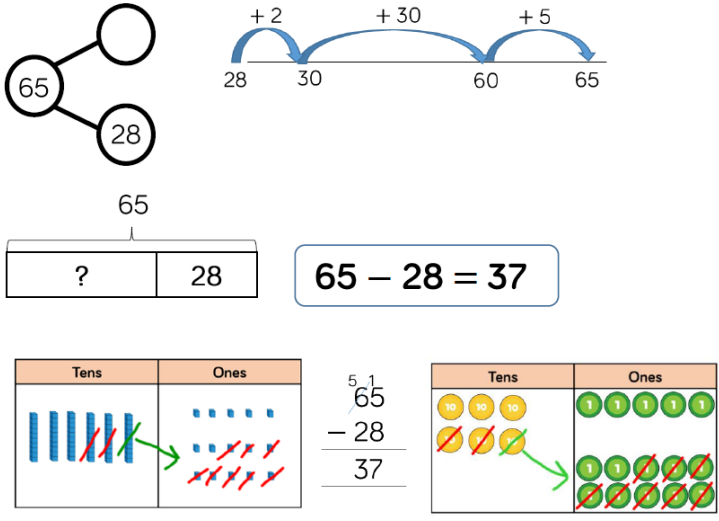
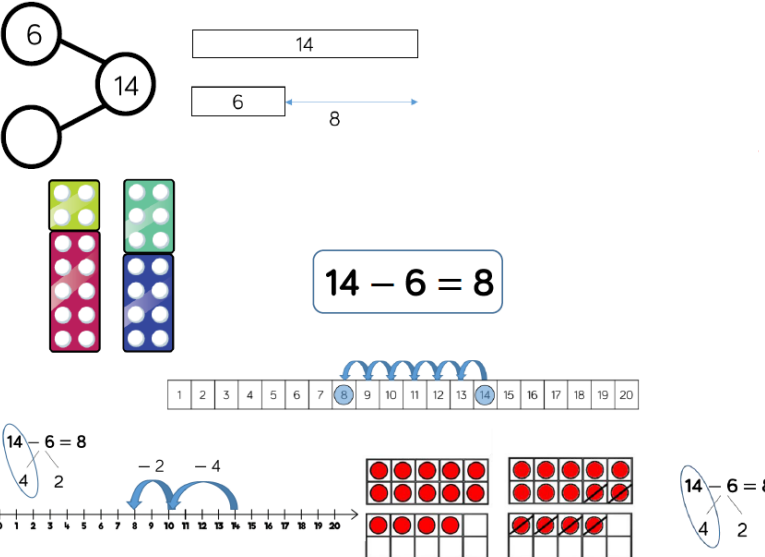
Subtraction



Subtraction representations used in Year 1 & 2

Skill	Year	Representations and models	
Subtract two 1-digit numbers to 10	1	Part-whole model Bar model Numicon	Ten frames (within 10) Number tracks
Subtract 1 and 2-digit numbers to 20	1	Part-whole model Bar model Numicon Ten frames (within 20)	Number tracks Number lines (labelled)
Subtract 1 and 2-digit numbers to 100	2	Part-whole model Bar model Number lines (labelled)	Number lines (blank) Hundred square
Subtract two 2-digit numbers	2	Part-whole model Bar model Number lines (blank)	Base 10 Place value counters

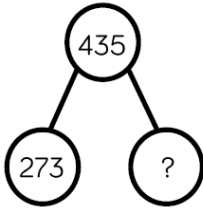
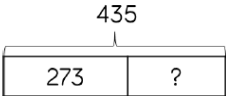
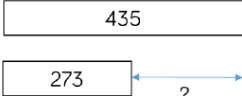
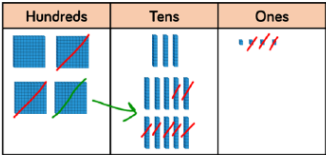
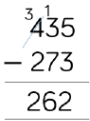
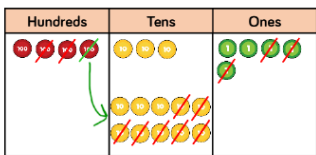
Subtraction representations used in Year 1 & 2

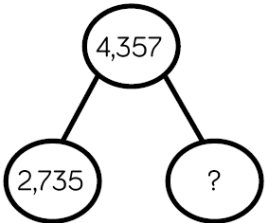
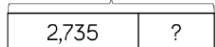
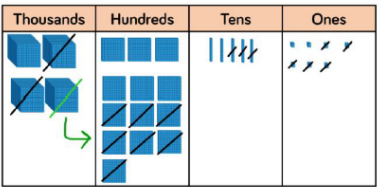
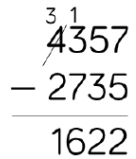
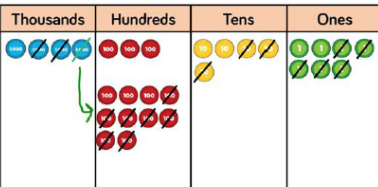
Skill: Subtract 1-digit numbers within 10	Year: 1	Skill: Subtract 1 and 2-digit numbers to 100	Year: 2
 $7 - 3 = 4$	Part-whole models, bar models, ten frames and number shapes support partitioning. Ten frames, number tracks, single bar models and bead strings support reduction. Cubes and bar models with two bars can support finding the difference.	 $65 - 28 = 37$	At this stage, encourage children to use the formal column method when calculating alongside straws, base 10 or place value counters. As numbers become larger, straws become less efficient. Children can also use a blank number line to count on to find the difference. Encourage them to jump to multiples of 10 to become more efficient.
Skill: Subtract 1 and 2-digit numbers to 20	Year: 1/2		
 $14 - 6 = 8$	When subtracting one-digit numbers that cross 10, it is important to highlight the importance of ten ones equalling one ten. Children should be encouraged to find the number bond to 10 when partitioning the subtracted number. Ten frames, number shapes and number lines are particularly useful for this.		

Subtraction representations used in Year 3 & 4

Skill	Year	Representations and models	
Subtract with up to 3-digits	3	Part-whole model Bar model	Base 10 Place value counters Column addition
Subtract with up to 4-digits	4	Part-whole model Bar model	Base 10 Place value counters Column addition

Subtraction representations used in Year 3 & 4

Skill: Subtract numbers with up to 3 digits	Year: 3
   $435 - 273 = 262$   	Base 10 and place value counters are the most effective manipulative when subtracting numbers with up to 3 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning.

Skill: Subtract numbers with up to 4 digits	Year: 4
    $4,357 - 2,735 = 1,622$   	Base 10 and place value counters are the most effective manipulatives when subtracting numbers with up to 4 digits. Ensure children write out their calculation alongside any concrete resources so they can see the links to the written column method. Plain counters on a place value grid can also be used to support learning.

Subtraction representations used in Year 5 & 6

Skill	Year	Representations and models	
Subtract with more than 4 digits	5	Part-whole model Bar model	Place value counters Column addition
Subtract with up to 3 decimal places	5	Part-whole model Bar model	Place value counters Column addition

Subtraction representations used in Year 5 & 6

Skill: Subtract numbers with more than 4 digits

Year: 5/6

The diagram illustrates the subtraction of 182,501 from 294,382. On the left, a tree diagram shows 294,382 branching into 182,501 and an unknown value (?). In the center, a grid represents the numbers: 294,382 is shown as 2 ten-thousands, 9 thousands, 4 hundreds, 3 tens, and 8 ones; 182,501 is shown as 1 ten-thousand, 8 thousands, 2 hundreds, 5 tens, and 1 one. To the right, a subtraction problem is shown: 294,382 minus 182,501 equals an unknown value (?).

$$294,382 - 182,501 = 111,881$$

Place value counters or plain counters on a place value grid are the most effective concrete resource when subtracting numbers with more than 4 digits.

At this stage, children should be encouraged to work in the abstract, using column method to subtract larger numbers efficiently.

The place value grid shows the subtraction of 182,501 from 294,382. The columns are labeled HTh, TTh, Th, H, T, and O. The grid contains counters for each number, with some counters crossed out to show the subtraction process. For example, in the HTh column, two red counters are crossed out. In the TTh column, two purple counters are crossed out. In the Th column, two blue counters are crossed out. In the H column, three green counters are crossed out. In the T column, two yellow counters are crossed out. In the O column, one red counter is crossed out.

	2	9	3	1	8	2
-	1	8	2	5	0	1
	1	1	1	8	8	1

Skill: Subtract with up to 3 decimal places

Year: 5

$$\begin{array}{r} 4 \quad 1 \\ 5.43 \\ - 2.7 \\ \hline 2.73 \end{array}$$

$$5.43 - 2.7 = 2.73$$

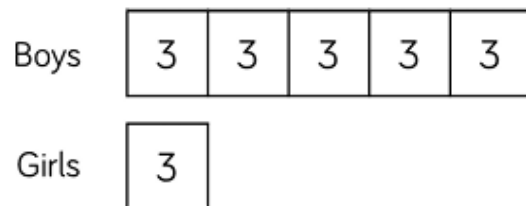
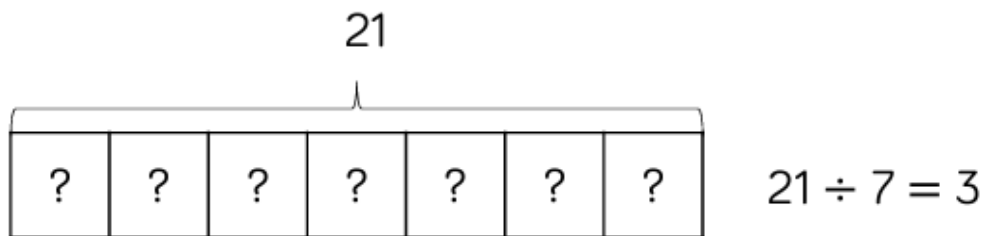
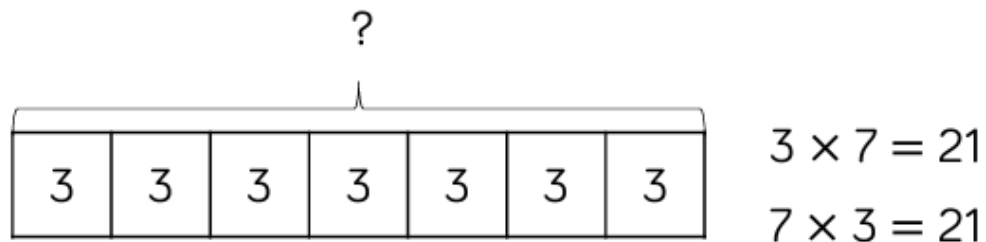
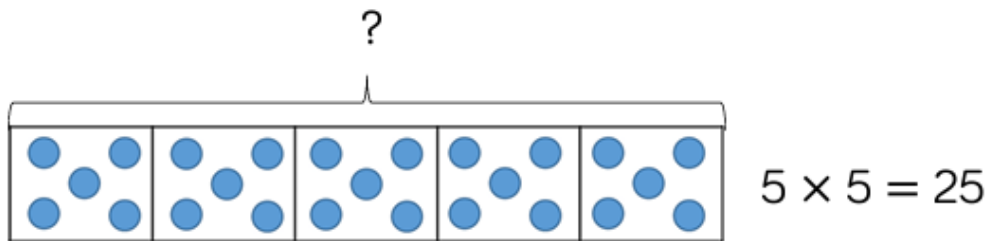
Place value counters and plain counters on a place value grid are the most effective manipulative when subtracting decimals with 1, 2 and then 3 decimal places.

Ensure children have experience of subtracting decimals with a variety of decimal places. This includes putting this into context when subtracting money and other measures.

Multiplication

X

Bar Model



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

The multiple bar model provides an opportunity to compare the groups.

Numicon



$$5 \times 4 = 20$$

$$4 \times 5 = 20$$

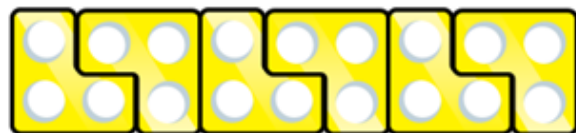


$$5 \times 4 = 20$$

$$4 \times 5 = 20$$



$$18 \div 3 = 6$$



Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd $\times$ odd = even, odd $\times$ even = odd, even $\times$ even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

Number Tracks



$$6 \times 3 = 18$$

$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

Benefits

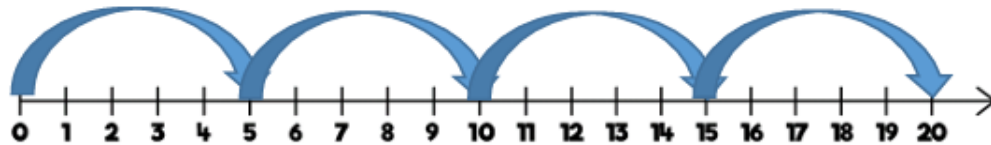
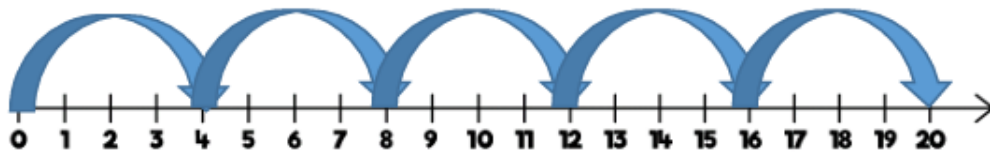
Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

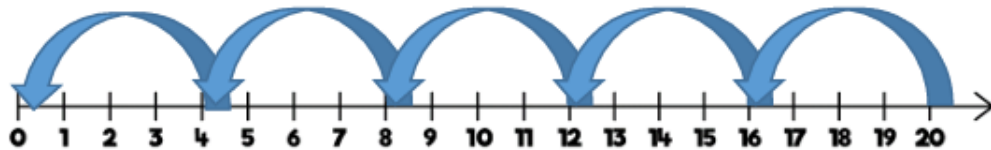
Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

Number Lines (labelled)



$$4 \times 5 = 20$$

$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

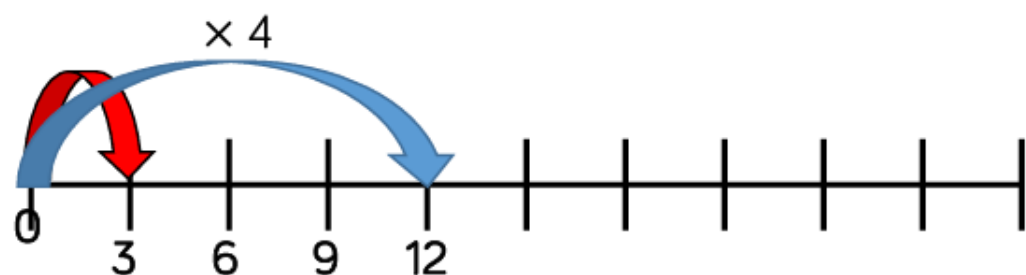
When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

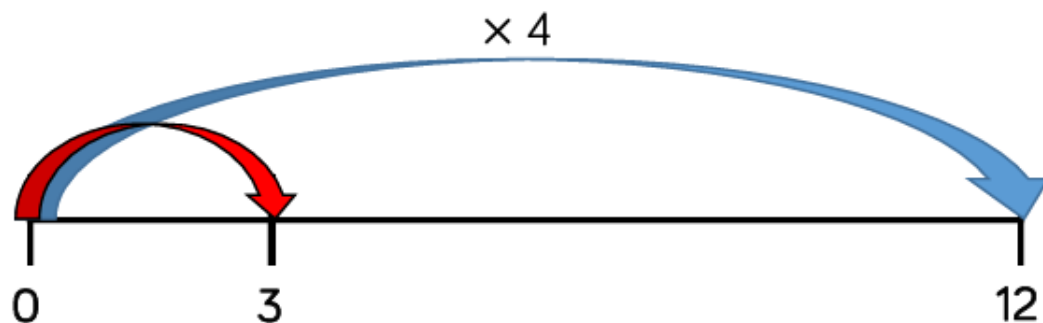
Number Lines (blank)



A red car travels 3 miles.

A blue car 4 times further.

How far does the blue car travel?



A blue car travels 12 miles.

A red car 4 times less.

How far does the red car travel?

Benefits

Children can use blank number lines to represent scaling as multiplication or division.

Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

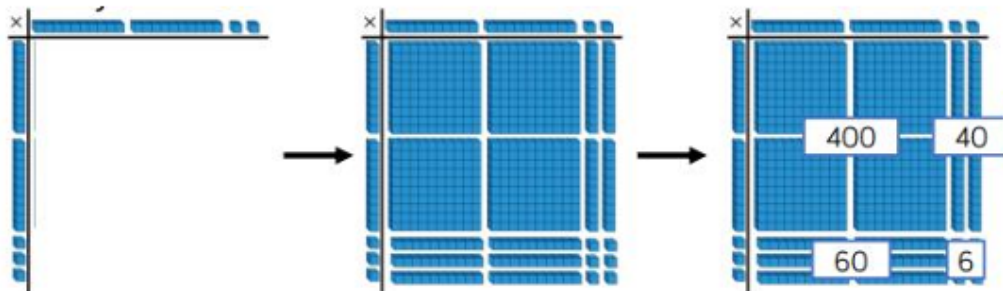
Blank number lines without intervals can also be used for children to represent scaling.

Base 10/Dienes (multiplication)

Hundreds	Tens	Ones
		●●●●
		●●●●
		●●●●

←

$$\begin{array}{r}
 24 \\
 \times 3 \\
 \hline
 72 \\
 \hline
 1
 \end{array}$$



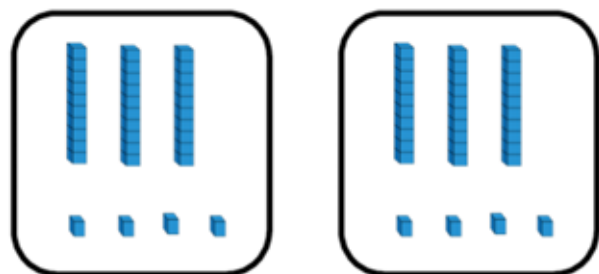
Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written representations match.

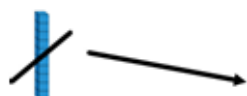
As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed.

Base 10 also supports the area model of multiplication well. Children use the equipment to build the number in a rectangular shape which they then find the area of by calculating the total value of the pieces. This area model can be linked to the grid method or the formal column method of multiplying 2-digits by 2-digits.

Base 10/Dienes (division)

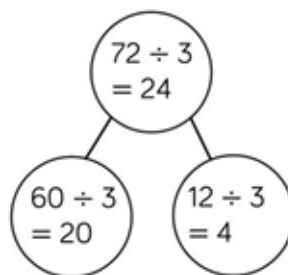


$$68 \div 2 = 34$$



Tens	Ones

$$72 \div 3 = 24$$



Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Place Value Counters (multiplication)

Hundreds	Tens	Ones
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
	10 10 10	1 1 1 1
100	10 10	

$$\begin{array}{r}
 34 \\
 \times 5 \\
 \hline
 170 \\
 \hline
 1 \ 2
 \end{array}$$

×	10 10 10 10	1 1 1 1
10	100 100 100 100	10 10 10 10
10	100 100 100 100	10 10 10 10
10	100 100 100 100	10 10 10 10
1	10 10 10 10	1 1 1 1
1	10 10 10 10	1 1 1 1

$$\begin{array}{r}
 44 \\
 \times 32 \\
 \hline
 8 \\
 80 \\
 120 \\
 + 1200 \\
 \hline
 1408 \\
 1
 \end{array}$$

Benefits

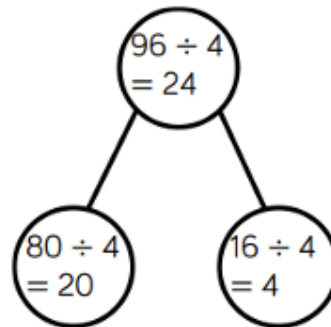
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed. The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Place Value Counters (division)

Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1
10 10	1 1 1 1
10 10	1 1 1 1



Thousands	Hundreds	Tens	Ones
1000 1000 1000 1000	100 100 100 100 100 100 100 100	10 10 10 10 10 10 10 10 10	1 1 1 1 1 1 1 1 1 1 1 1

$$\begin{array}{r} 1223 \\ 4 \overline{) 4892} \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of division.

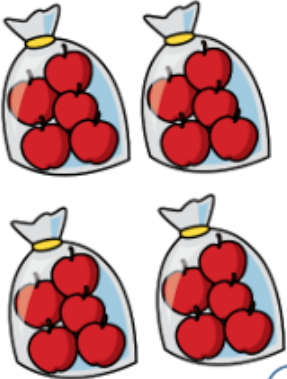
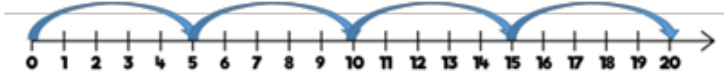
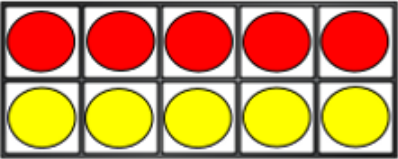
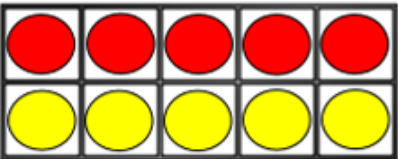
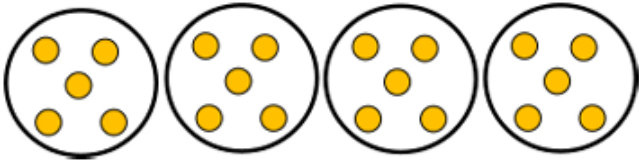
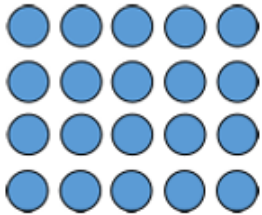
When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

Multiplication representations used in Year 1 & 2

Skill	Year	Representations and models	
Solve one-step problems with multiplication	1/2	Bar model Numicon Counters	Ten frames Number lines

Multiplication representations used in Year 1 & 2

Skill: Solve 1-step problems using multiplication	Year: 1/2
   One bag holds 5 apples. How many apples do 4 bags hold?     $5 + 5 + 5 + 5 = 20$$4 \times 5 = 20$$5 \times 4 = 20$	Children represent multiplication as repeated addition in many different ways. In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record multiplication formally. In Year 2, children are introduced to the multiplication symbol.

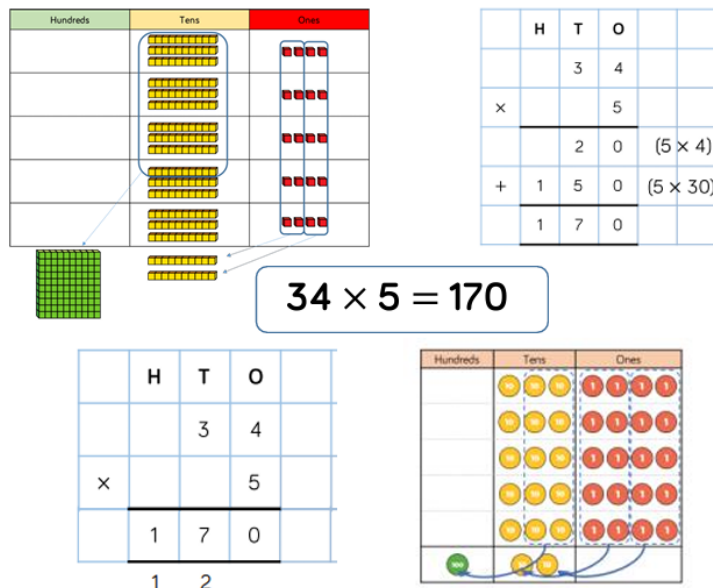
Multiplication representations used in Year 3 & 4

Skill	Year	Representations and models	
Multiply 2-digit by 1-digit numbers	3/4	Place value counters Base 10	Expanded written method Short written method
Multiply 3-digit by 1-digit numbers	4	Place value counters Base 10	Short written method

Multiplication representations used in Year 3 & 4

Skill: Multiply 2-digit numbers by 1-digit numbers

Year: 3/4

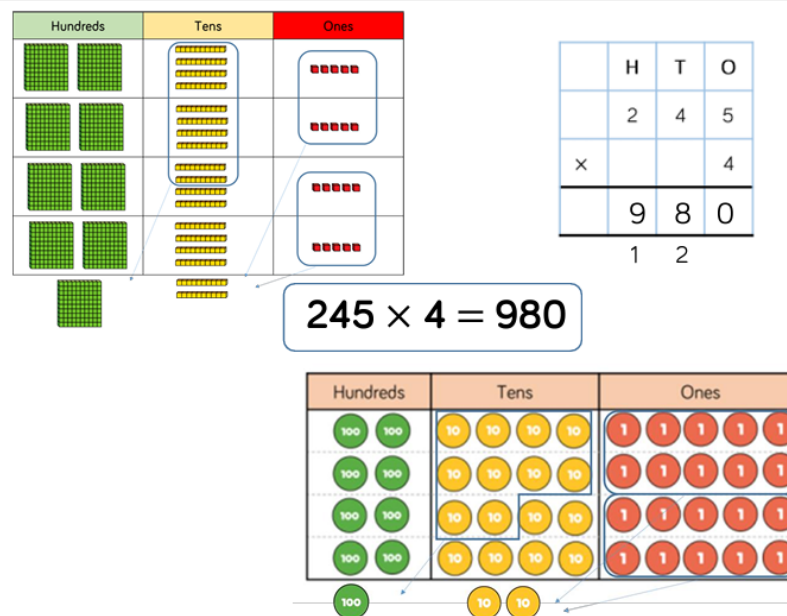


$$34 \times 5 = 170$$

Informal methods and the expanded method are used in Year 3 before moving on to the short multiplication method in Year 4. Place value counters should be used to support the understanding of the method rather than supporting the multiplication, as children should use times table knowledge.

Skill: Multiply 3-digit numbers by 1-digit numbers

Year: 4



$$245 \times 4 = 980$$

When moving to 3-digit by 1-digit multiplication, encourage children to move towards the short, formal written method. Base 10 and place value counters continue to support the understanding of the written method. Limit the number of exchanges needed in the questions and move children away from resources when multiplying larger numbers.

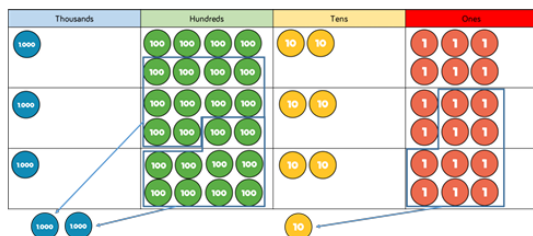
Multiplication representations used in Year 5 & 6

Skill	Year	Representations and models	
Multiply 4-digit by 1-digit numbers	5	Place value counters	Short written method
Multiply 2-digit by 2-digit numbers	5	Place value counters Base 10	Short written method Grid method
Multiply 2-digit by 3-digit numbers	5	Place value counters	Short written method Grid method
Multiply 2-digit by 4-digit numbers	5/6	Formal written method	

Multiplication representations used in Year 5 & 6

Skill: Multiply 4-digit numbers by 1-digit numbers

Year: 5



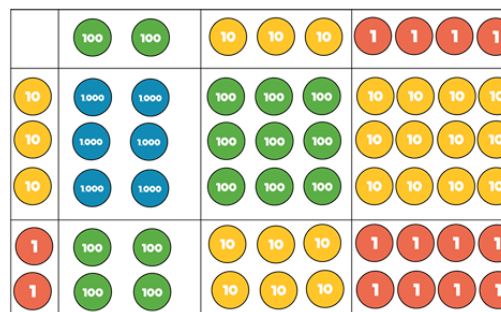
$$1,826 \times 3 = 5,478$$

	Th	H	T	O
	1	8	2	6
x				3
	5	4	7	8
	2		1	

When multiplying 4-digit numbers, place value counters are the best manipulative to use to support children in their understanding of the formal written method. If children are multiplying larger numbers and struggling with their times tables, encourage the use of multiplication grids so children can focus on the use of the written method.

Skill: Multiply 3-digit numbers by 2-digit numbers

Year: 5



Th	H	T	O
	2	3	4
x		3	2
	4	6	8
1	7	0	2
7	4	8	8

x	200	30	4
30	6,000	900	120
2	400	60	8

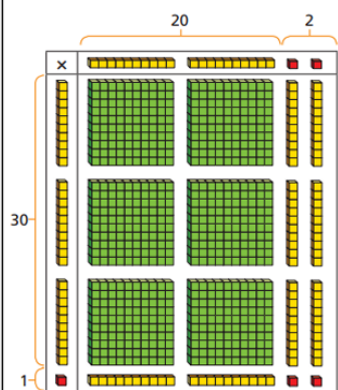
$$234 \times 32 = 7,488$$

Children can continue to use the area model when multiplying 3-digits by 2-digits. Place value counters become more efficient to use but Base 10 can be used to highlight the size of numbers.

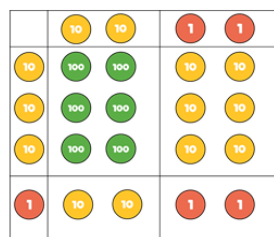
Children should now move towards the formal written method, seeing the links with the grid method.

Skill: Multiply 2-digit numbers by 2-digit numbers

Year: 5



$$22 \times 31 = 682$$



x	20	2
30	600	60
1	20	2

	H	T	O
		2	2
x		3	1
		2	2
	6	6	0
	6	8	2

When multiplying a multi-digit number by 2-digits, use the area model to help children understand the size of the numbers they are using. This links to finding the area of a rectangle by finding the space covered by the Base 10. The grid method matches the area model as an initial written method before moving on to the formal written multiplication method.

Skill: Multiply 4-digit numbers by 2-digit numbers

Year: 5/6

TTh	Th	H	T	O
	2	7	3	9
x			2	8
	2	1	9	1
2	5	3	7	2
5	4	7	8	0
1		1		
7	6	6	9	2

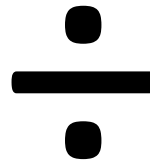
$$2,739 \times 28 = 76,692$$

When multiplying 4-digits by 2-digits, children should be confident in using the formal written method.

If they are still struggling with times tables, provide multiplication grids to support when they are focusing on the use of the method.

Consider where exchanged digits are placed and make sure this is consistent.

Division



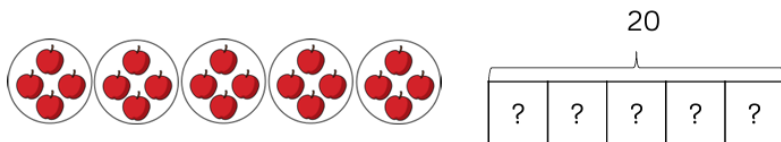
Division representations used in Year 1 & 2

Skill	Year	Representations and models	
Solve one-step problems with division (sharing)	1/2	Bar model Real life objects	Arrays Counters
Solve one-step problems with division (grouping)	1/2	Real life objects Numicon Ten frames	Number lines Arrays Counters

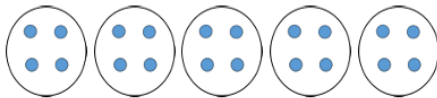
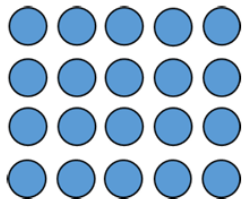
Division representations used in Year 1 & 2

Skill: Solve 1-step problems using multiplication (sharing)

Year: 1/2



There are 20 apples altogether.
They are shared equally between 5 bags.
How many apples are in each bag?



$$20 \div 5 = 4$$

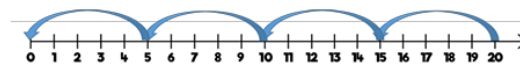
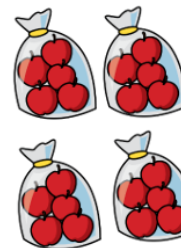
Children solve problems by sharing amounts into equal groups.

In Year 1, children use concrete and pictorial representations to solve problems. They are not expected to record division formally.

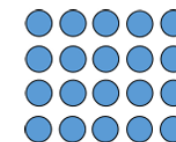
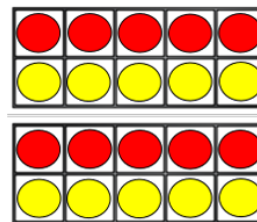
In Year 2, children are introduced to the division symbol.

Skill: Solve 1-step problems using division (grouping)

Year: 1/2



There are 20 apples altogether.
They are put in bags of 5.
How many bags are there?



$$20 \div 5 = 4$$

Children solve problems by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction on a number line. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division.

Division representations used in Year 3 & 4

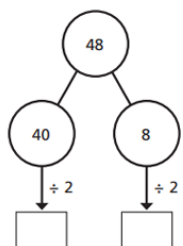
Skill	Year	Representations and models	
Divide 2-digits by 1-digit (no exchange sharing)	3	Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (sharing with exchange)	3	Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (sharing with remainders)	3/4	Base 10 Bar model	Place value counters Part-whole model
Divide 2-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division
Divide 3-digits by 1-digit (sharing with exchange)	4	Base 10 Bar model	Place value counters Part-whole model
Divide 3-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division

Division representations used in Year 3 & 4

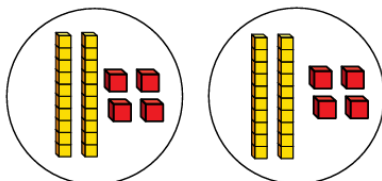
Skill: Divide 2-digits by 1-digit (sharing with no exchange)

Year: 3

Tens	Ones
10 10	1 1 1 1
10 10	1 1 1 1



$$48 \div 2 = 24$$



When dividing larger numbers, children can use manipulatives that allow them to partition into tens and ones.

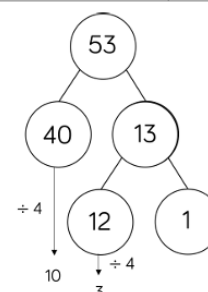
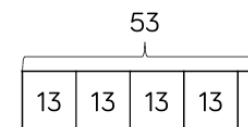
Straws, Base 10 and place value counters can all be used to share numbers into equal groups.

Part-whole models can provide children with a clear written method that matches the concrete representation.

Skill: Divide 2-digits by 1-digit (sharing with remainders)

Year: 3/4

Tens	Ones
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1



$$53 \div 4 = 13 \text{ r}1$$

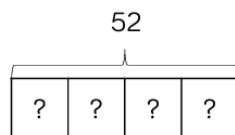
Tens	Ones
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1

When dividing numbers with remainders, children can use Base 10 and place value counters to exchange one ten for ten ones. Starting with the equipment outside the place value grid will highlight remainders, as they will be left outside the grid once the equal groups have been made. Flexible partitioning in a part-whole model supports this method.

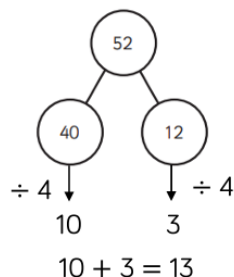
Skill: Divide 2-digits by 1-digit (sharing with exchange)

Year: 3/4

Tens	Ones
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1



$$52 \div 4 = 13$$



Tens	Ones
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1
10 10 10 10	1 1 1 1 1 1 1 1

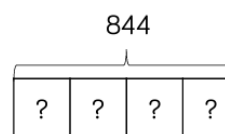
When dividing numbers involving an exchange, children can use Base 10 and place value counters to exchange one ten for ten ones. Children should start with the equipment outside the place value grid before sharing the tens and ones equally between the rows.

Flexible partitioning in a part-whole model supports this method.

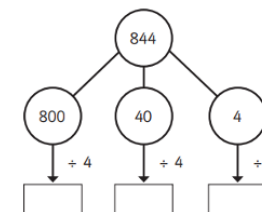
Skill: Divide 3-digits by 1-digit (sharing)

Year: 4

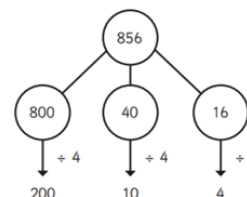
$$844 \div 4 = 211$$



H	T	O
100 100 100 100	10 10 10 10	1 1 1 1 1 1 1 1
100 100 100 100	10 10 10 10	1 1 1 1 1 1 1 1
100 100 100 100	10 10 10 10	1 1 1 1 1 1 1 1
100 100 100 100	10 10 10 10	1 1 1 1 1 1 1 1



$$856 \div 4 = 214$$



Hundreds	Tens	Ones
100 100 100 100	10 10 10 10	1 1 1 1 1 1 1 1
100 100 100 100	10 10 10 10	1 1 1 1 1 1 1 1
100 100 100 100	10 10 10 10	1 1 1 1 1 1 1 1
100 100 100 100	10 10 10 10	1 1 1 1 1 1 1 1

Children can continue to use place value counters to share 3-digit numbers into equal groups. Children should start with the equipment outside the place value grid before sharing the hundreds, tens and ones equally between the rows. This method can also help to highlight remainders. Flexible partitioning in a part-whole model supports this method.

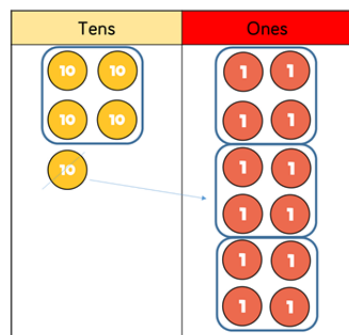
Division representations used in Year 5 & 6

Skill	Year	Representations and models	
Divide 3-digits by 1-digit (grouping)	4/5	Place value counters Counters	Place value grid Written short division
Divide 4-digits by 1-digit (grouping)	5	Place value counters Counters	Place value grid Written short division
Divide multi-digits by 2-digits (short division)	6	Written short division	List of multiples
Divide multi-digits by 2-digits (long division)	6	Written long division	List of multiples

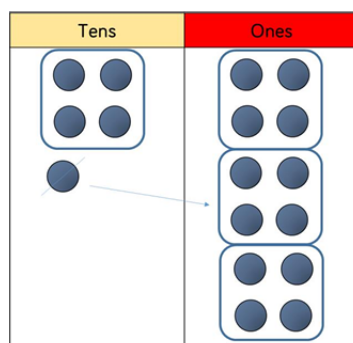
Division representations used in Year 5 & 6

Skill: Divide 2-digits by 1-digit (grouping)

Year: 5



		1	3	
	4	5	1	2



$$52 \div 4 = 13$$

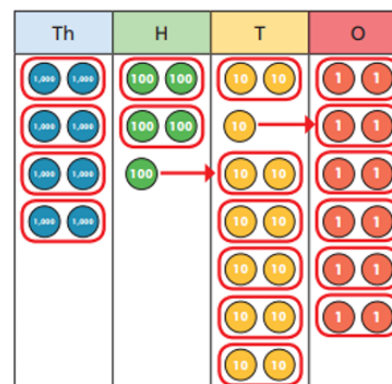
When using the short division method, children use grouping. Starting with the largest place value, they group by the divisor.

Language is important here. Children should consider 'How many groups of 4 tens can we make?' and 'How many groups of 4 ones can we make?'

Remainders can also be seen as they are left ungrouped.

Skill: Divide 4-digits by 1-digit (grouping)

Year: 5



		4	2	6	6
2	8	5	1	3	1

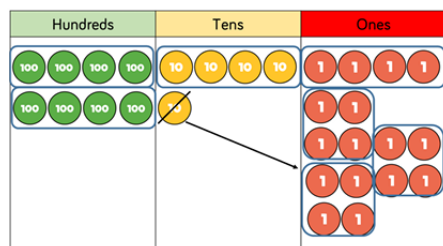
$$8,532 \div 2 = 4,266$$

Place value counters or plain counters can be used on a place value grid to support children to divide 4-digits by 1-digit. Children can also draw their own counters and group them through a more pictorial method.

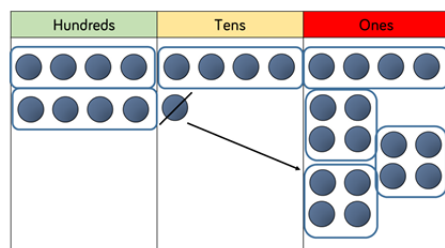
Children should be encouraged to move away from the concrete and pictorial when dividing numbers with multiple exchanges.

Skill: Divide 3-digits by 1-digit (grouping)

Year: 5



		2	1	4
	4	8	5	1



$$856 \div 4 = 214$$

Children can continue to use grouping to support their understanding of short division when dividing a 3-digit number by a 1-digit number.

Place value counters or plain counters can be used on a place value grid to support this understanding. Children can also draw their own counters and group them through a more pictorial method.

Skill: Divide multi digits by 2-digits (short division)

Year: 6

		0	3	6
	12	4	4	7
		3	2	

$$432 \div 12 = 36$$

$$7,335 \div 15 = 489$$

		0	4	8	9
15	7	7	13	13	5
		3	3	5	

15	30	45	60	75	90	105	120	135	150
----	----	----	----	----	----	-----	-----	-----	-----

When children begin to divide up to 4-digits by 2-digits, written methods become the most accurate as concrete and pictorial representations become less effective. Children can write out multiples to support their calculations with larger remainders. Children will also solve problems with remainders where the quotient can be rounded as appropriate.